Semester-Fahrplan

Datum	V	Thema
13.04.2018	1	Einführung / Qt Installation
20.04.2018	3	Grundlagen: Kontinuumsmechanik
27.04.2018	5	Grundlagen: Hydromechanik
04.05.2018	7	Grundlagen: Partielle Differentialgleichungen
11.05.2018	9	Grundlagen: Numerik, Qt Übung: Funktionsrechner
18.05.2018	11	Numerik: Finite Differenzen Methode I (explizit)
01.06.2018	13	Numerik: Finite Differenzen Methode II (implizit)
08.06.2018	15	Gerinnehydraulik: Theorie – Grundlagen
15.06.2018	17	Gerinnehydraulik: Programmierung, Übung 1
22.06.2018	19	Gerinnehydraulik: Programmierung, Übung 2
29.06.2018	21	Grundwassermodellierung: Catchment Übung
06.07.2018	23	Grundwassermodellierung: Datenbasierte Methoden I
13.07.2018	25	Grundwassermodellierung: Datenbasierte Methoden II
20.07.2018	27	Klausurvorbereitung

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Hydroinformatik II
"Prozesssimulation und Systemanalyse"
Grundlagen Kontinuumsmechanik

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[2] Centre for Advanced Water Research – CAWR

20./27.04.2018 - Dresden

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Fahrplan für heute ...

- Motivation
- Lagrange Konzept
- Euler Konzept
- Reynolds Transport Theorem
- Fluxes
- Bilanzgleichungen
- Erhaltungsgrößen

$\Longrightarrow$ viel Theorie - vor allem die mathematische Schreibweise verstehen ”zu lesen”

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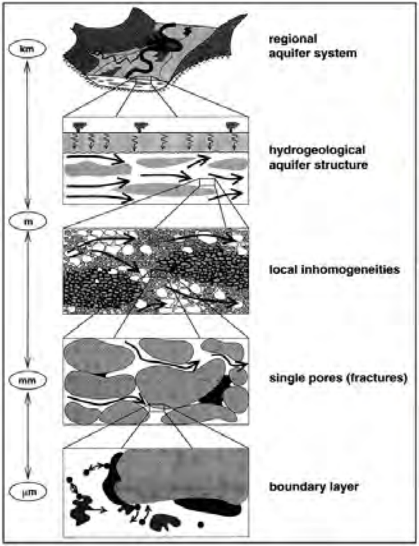
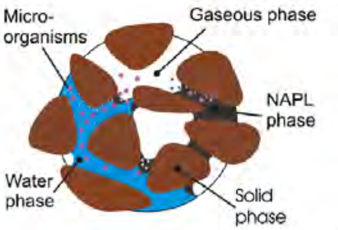
Skalen





Quellen: Kobus et al. (1995), Kolditz (2002)

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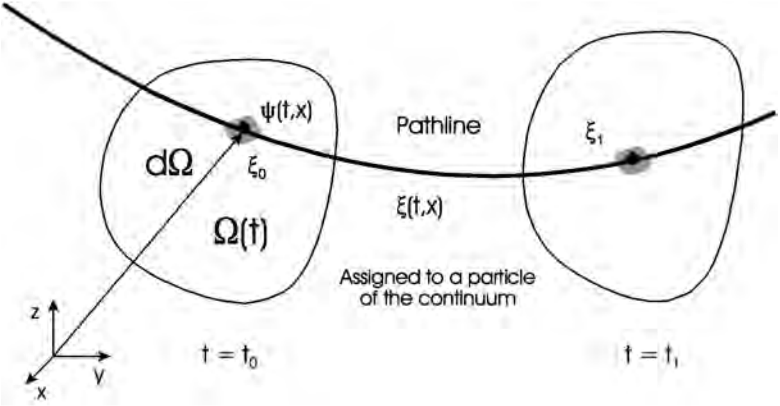
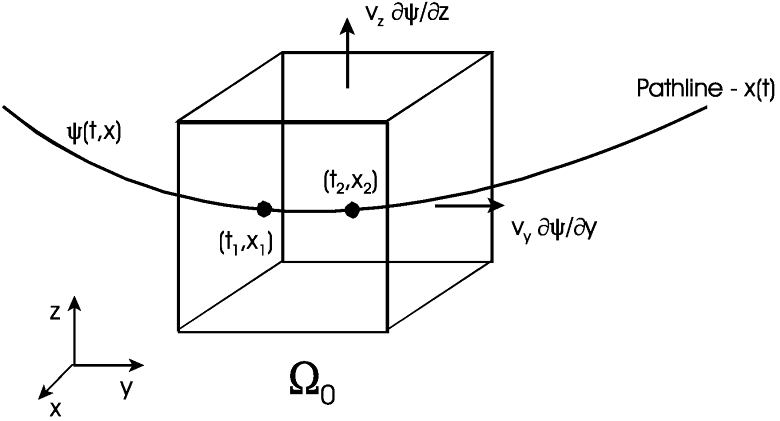
Lagrange Konzept (1.1.1)



Euler Konzept (1.1.1)



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20.04.2018

- Volumenintegral (Zeichnung)

$$\int_{\Omega} d\Omega \tag{1}$$

$$\int_a^b f(x)dx = \lim_{(x_{k+1}-x_k)\to 0} \sum_{k=1}^{\infty} (f(x_{k+1} - f(x_k)) (x_{k+1} - x_k) \tag{2}$$

- Oberflächen-(Ring)-Integral (Zeichnung)

$$\oint_{\partial\Omega} \mathbf{n} \cdot d\mathbf{S} \tag{3}$$

- Materielle Ableitung

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v} \cdot \nabla\psi \tag{4}$$

- Gradient (Vektor)

$$\nabla = \{\partial/\partial x, \partial/\partial y, \partial/\partial z\} \tag{5}$$

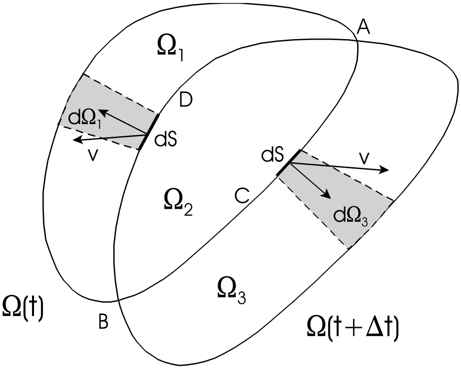
- Divergenz (Skalar)

$$\nabla \cdot \mathbf{v} = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z \tag{6}$$

Aufgabe: Was ist $\mathbf{v} \cdot \nabla\psi$?

Reynolds Transport Theorem (Lagrange) (1.1.3)

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \int_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \oint_{\partial\Omega}\psi(t)\mathbf{v}\cdot d\mathbf{S} = \int_{\Omega}q^{\psi}d\Omega \qquad (7)$$



Beweisführung:
Siehe Skript
Abschn. 1.1.3

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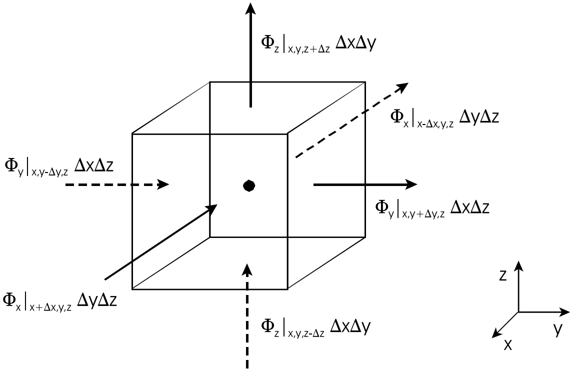
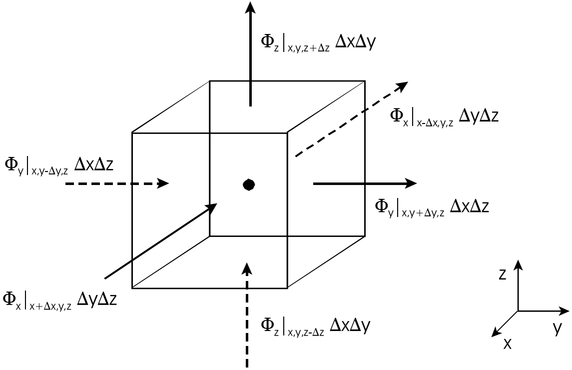


$$\Phi_x^\psi = \partial\psi/\partial x \quad , \quad \Phi^\psi = \nabla\psi$$

Frage: Ist Φ^ψ eine skalare oder vektorielle Größe ?

Reynolds Transport Theorem (Euler) (1.1.3)



$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \qquad (8)$$

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$$\oint_{\partial\Omega} \Phi^{\psi} \cdot d\mathbf{S} = \int_{\Omega} \nabla \cdot \Phi^{\psi} d\Omega \tag{9}$$

$$\lim_{\Omega \to 0} \frac{1}{\Omega} \oint_{\partial\Omega} \Phi \cdot d\mathbf{S} = \nabla \cdot \Phi \tag{10}$$

Point-Flux

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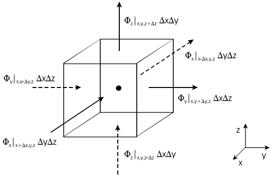

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \underbrace{\frac{\partial}{\partial t}\int_{\Omega}\psi d\Omega}_{1} + \underbrace{\oint_{\partial\Omega}\Phi^{\psi}\cdot d\mathbf{S}}_{2} = \underbrace{\int_{\Omega}q^{\psi}d\Omega}_{3} \qquad (11)$$

with:

1. Rate of change of total amount of quantity ψ in the control volume,
2. Net rate of increase / decrease of ψ due to fluxes,
3. Rate of increase / decrease of ψ due to sources.

Reynolds Transport Theorem (Euler) (1.1.3)

$$\frac{d}{dt}\int\limits_{\Omega}\psi d\Omega = \underbrace{\frac{\partial}{\partial t}\int\limits_{\Omega}\psi d\Omega}_{1} + \underbrace{\oint\limits_{\partial\Omega}\Phi^{\psi}\cdot d\mathbf{S}}_{2} = \underbrace{\int\limits_{\Omega} q^{\psi} d\Omega}_{3} \qquad (12)$$

using

$$\oint\limits_{\partial\Omega}\Phi^{\psi}\cdot d\mathbf{S} = \int\limits_{\Omega}\nabla\cdot\Phi^{\psi} d\Omega \qquad (13)$$

$$\frac{d}{dt}\int\limits_{\Omega}\psi d\Omega = \int\limits_{\Omega}\frac{\partial\psi}{\partial t} d\Omega + \int\limits_{\Omega}\nabla\cdot\Phi^{\psi} d\Omega = \int\limits_{\Omega} q^{\psi} d\Omega \qquad (14)$$

Reynolds Transport Theorem

$$\frac{d}{dt}\int_{\Omega}\psi d\Omega = \int_{\Omega}\frac{\partial\psi}{\partial t}d\Omega + \int_{\Omega}\nabla\cdot\Phi^{\psi}d\Omega = \int_{\Omega}q^{\psi}d\Omega \qquad (15)$$

$$\forall\Omega : \frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla\cdot\Phi^{\psi} = q^{\psi} \qquad (16)$$

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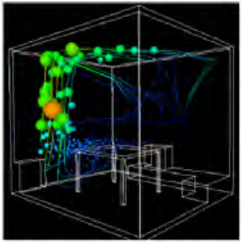
Reynolds Transport Theorem

$$\frac{d}{dt}\int\limits_{\Omega} \psi \, d\Omega = \int\limits_{\Omega} \left(\frac{\partial \psi}{\partial t} + \nabla \cdot \mathbf{\Phi}^{\psi} \right) d\Omega = \int\limits_{\Omega} q^{\psi} \, d\Omega$$

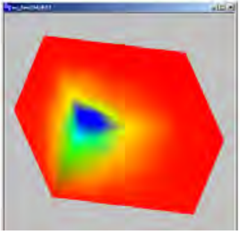
$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + \mathbf{v}^{E} \nabla \psi$$

Lagrange

Euler



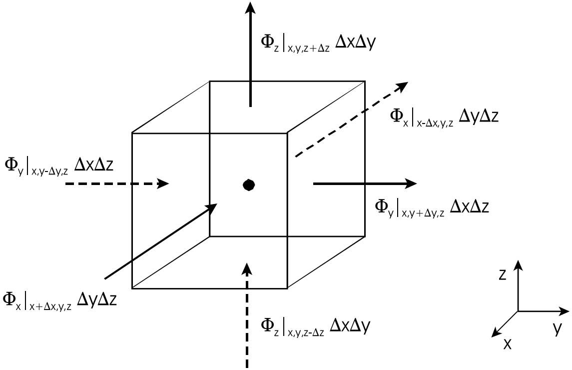
http://www.cscs.ch/~mvalle/Libro/



Fluxes (1.1.6)

The total flux Φ^{ψ} of a quantity ψ is defined as

$$\Phi^{\psi} = \mathbf{v}^{E}\psi \qquad (17)$$



Fluxes (1.1.6)

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot \Phi^{\psi} = q^{\psi} \tag{18}$$

$$\Phi^{\psi} = \mathbf{v}^{E}\psi \tag{19}$$

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla \cdot (\mathbf{v}^{E}\psi) = q^{\psi} \tag{20}$$

Fluxes (1.1.6)

$$\Phi^{\psi} = \mathbf{v}^E \psi = \underbrace{\mathbf{v}\psi}_{\Phi_A^{\psi}} + \underbrace{(\mathbf{v}^E - \mathbf{v})\psi}_{\Phi_D^{\psi}} \tag{21}$$

and, therefore, decomposed into two parts: an advective flux Φ_A^{ψ} and a diffusive flux Φ_D^{ψ} relative to the mass-weighted velocity:

- advective flux of quantity ψ

$$\Phi_A^{\psi} = \mathbf{v}\psi \tag{22}$$

- diffusive flux of quantity ψ (Fick's law)

$$\Phi_D^{\psi} = -\mathbf{D}^{\psi}\nabla\psi \tag{23}$$

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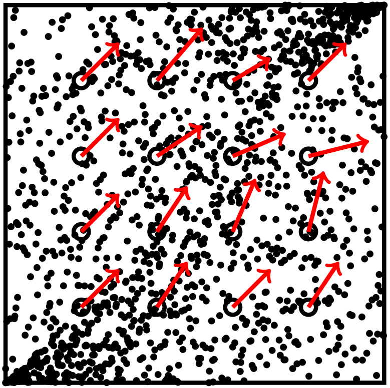

$$\mathbf{v}^E = \cup \mathbf{v}_i$$

$$\mathbf{v}^E = \mathbf{v} + \hat{\mathbf{v}}$$

$$\hat{\mathbf{v}} = \mathbf{v}^E - \mathbf{v}$$

General Balance Equation (1.1.7)

- Integral form

$$\int_\Omega \frac{d\psi}{dt} = \int_\Omega \frac{\partial\psi}{\partial t} + \int_\Omega \nabla\cdot(\mathbf{v}\psi) - \int_\Omega \nabla\cdot(\mathbf{D}^\psi\nabla\psi) = \int_\Omega Q^\psi \tag{24}$$

- Differential form

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \nabla\cdot(\mathbf{v}\psi) - \nabla\cdot(\mathbf{D}^\psi\nabla\psi) = Q^\psi \tag{25}$$

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Conservation Quantities (1.1.2)

The amount of a quantity in a defined volume Ω is given by

$$\Psi = \int_{\Omega} \psi d\Omega(t) \tag{26}$$

where Ψ is an extensive conservation quantity (i.e. mass, momentum, energy) and ψ is the corresponding intensive conservation quantity such as mass density ρ, momentum density $\rho\mathbf{v}$ or energy density e.

Extensive quantity	Symbol	Intensive quantity	Symbol
Mass	M	Mass density	ρ
Linear momentum	$\mathbf{m}$	Linear momentum density	$\rho\mathbf{v}$
Energy	E	Energy density	$e = \rho i + \frac{1}{2}\rho v^2$

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