

Modellierung von Hydrosystemen
"Numerische und daten-basierte Methoden"
BHYWI-22-V2 @ 2019
Einführung und Wdh. Mechanik

Olaf Kolditz

*Helmholtz Centre for Environmental Research – UFZ

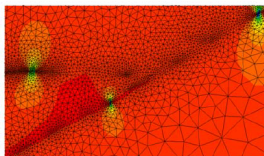
¹Technische Universität Dresden – TUDD

²Centre for Advanced Water Research – CAWR

12.04.2019 - Dresden

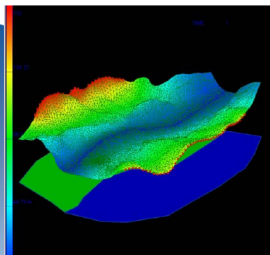
- ▶ Grundlagen (kurze Wiederholung)
- ▶ Grundwassergleichung
- ▶ Prinzip-Beispiel
- ▶ Bilanzierung
- ▶ Berechnungsverfahren
- ▶ Lösung
- ▶ Übung MvH-01: Programmierung für Prinzip-Beispiel

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + \mathbf{v}^E \nabla\psi$$

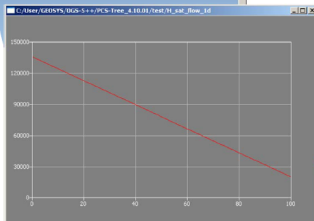
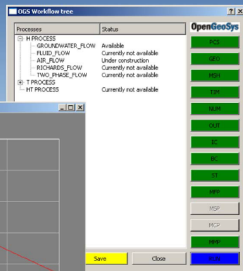


Basics
Mechanik

Anwendung

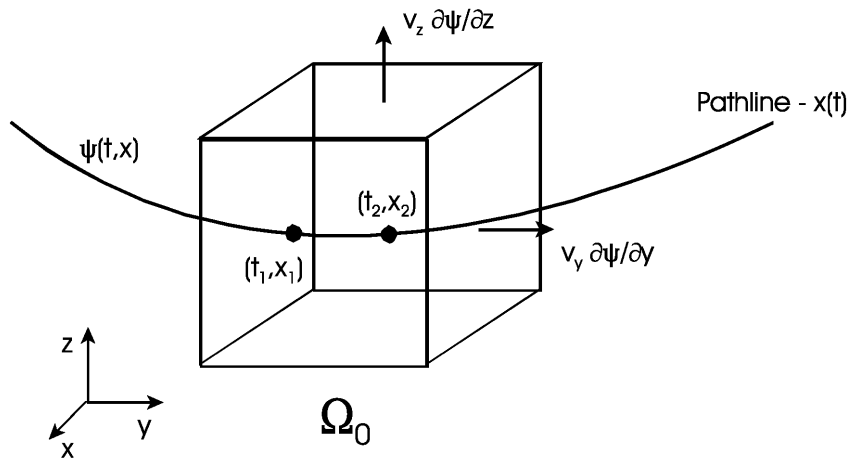


Numerische
Methoden



Programmierung
Visual C++

Das Euler Prinzip (Wdh)



$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f}^e + \frac{1}{\rho} \nabla \cdot \sigma \quad (1)$$

In index notation the above vector equation is written as

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= \frac{1}{\rho} \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} \right) \\ \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= \frac{1}{\rho} \left(\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} \right) \\ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= g + \frac{1}{\rho} \left(\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) \end{aligned} \quad (2)$$

with $u = v_x$, $v = v_y$, $w = v_z$ and $\mathbf{f}^e = \mathbf{g}$.

Flow Equations - Systematic (Wdh)

Stress Tensor

$$\boldsymbol{\sigma} = -p\mathbf{1} + \boldsymbol{\tau} \quad (3)$$

Navier-Stokes Equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f}^e - \frac{1}{\rho} \nabla p + \nu \Delta \mathbf{v} \quad (4)$$

Euler Equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \mathbf{f}^e - \frac{1}{\rho} \nabla p \quad (5)$$

Stokes Equation

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{f}^e - \frac{1}{\rho} \nabla p + \nu \Delta \mathbf{v} \quad (6)$$

Darcy Equations

$$0 = \mathbf{f}^e - \frac{1}{\rho} \nabla p + \nu \Delta \mathbf{v}$$

$$\frac{\partial n\rho}{\partial t} + \nabla \cdot (n\rho\mathbf{v}) = Q_\rho \quad (8)$$

Für ein inkompressibles Fluid gilt dann (PF)

$$\rho \frac{\partial n}{\partial t} + \rho \nabla \cdot (n\mathbf{v}) = Q_\rho \quad (9)$$

oder noch besser

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = \frac{Q_\rho}{\rho_0} \quad (10)$$

In der Grundwasserhydraulik gilt

$$\frac{\partial n}{\partial t} = S \frac{\partial h}{\partial t} \quad (11)$$
$$n\mathbf{v} = \mathbf{q} = -\mathbf{K}\nabla h \quad (\text{Darcy Gesetz}) \quad (12)$$

Dabei sind: S der Speicherkoeffizient, h die Piezometer- oder hydraulische Höhe, \mathbf{q} die Darcy- oder Filtergeschwindigkeit und \mathbf{K} der hydraulische Leitfähigkeitstensor.

$$S \frac{\partial h}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q$$

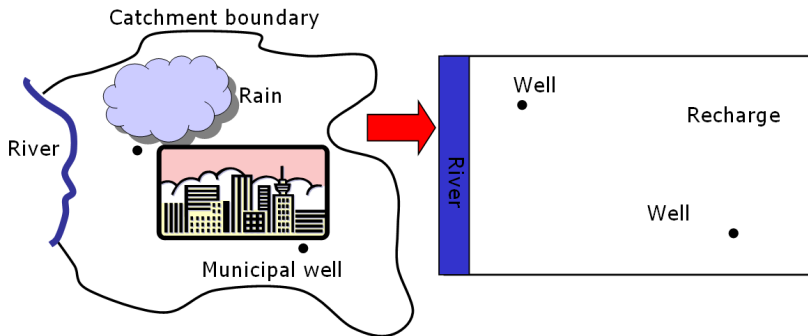
$$S \frac{\partial h}{\partial t} - \nabla \cdot (\mathbf{K} \nabla h) = Q$$

$$S \frac{\partial h}{\partial t} - \frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) - \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) - \frac{\partial}{\partial z} \left(K_z \frac{\partial h}{\partial z} \right) = Q$$

Wir begnügen uns mit einem 2-D horizontalen Modell.

$$S \frac{\partial h}{\partial t} - \frac{\partial}{\partial x} \left(K_x \frac{\partial h}{\partial x} \right) - \frac{\partial}{\partial y} \left(K_y \frac{\partial h}{\partial y} \right) = Q \quad (13)$$

Prinzip-Beispiel



Quelle: Sebastian Bauer (Uni Kiel)