Darcy Experiment

It is the year 1857. Keen researchers have set up a novel experiment to understand flow through porous media and associated pressure loss. Originally given in French, this is a rough translation of one of the researchers' diary entries.

Flow direction experiments

Entry from July, 4th, 1857. Dear diary, I am very excited today. I wasn't even able to finish the blueberry cake my wife made... I need to tell you first, that we finally managed to set up a number of experiments from which I am sure that they will be a tremendous breakthrough for our research.

We constructed an apparatus that works as follows. A cylindrical column was filled with soil from my backyard. When filling the soil into the column, we took great care not to introduce any layers, so the porous medium should be homogeneous. The column has an inflow and an outflow port which are connected to two tubes which allow us to push water through the column. At both tubes (in and outflow), we added stand pipes that allow us to measure the water levels. I added some sketches of the conceptual setups in the end (figure 1).

I am still wondering about the flow direction in the column, though... is the water always flowing from the high to the low water level, even if it is then raising from the bottom to the top of the cylinder? Can this really be true? Maybe, some day, I will understand this...

> **DIY:** *Help the researcher and use your magic time-travel-pen and add arrows to show flow directions in the columns!*

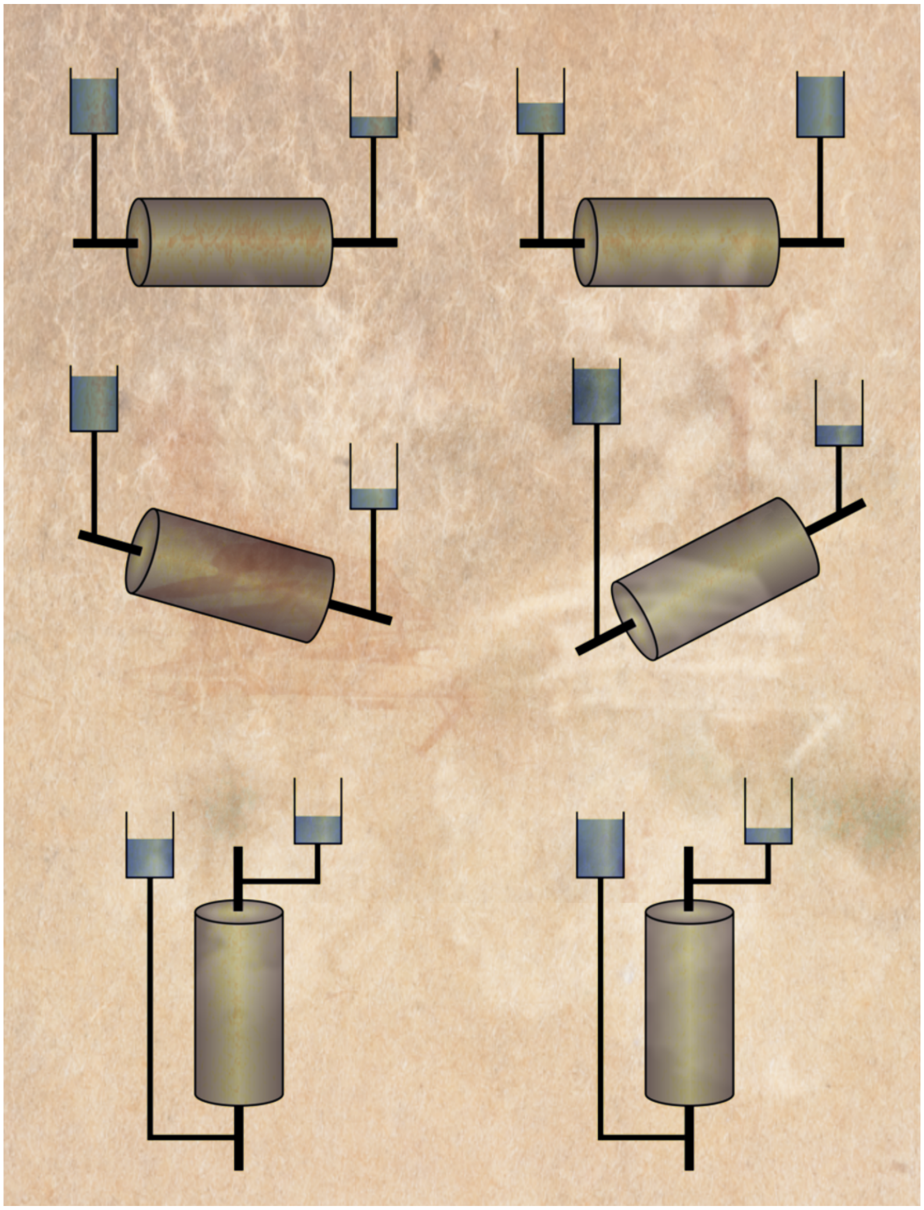

Figure 1: Sketches of column experiments.

Flow calculations

Entry from August, 18th, 1857. Dear diary, this story is getting even better. After contemplating day and night for many weeks, I think that we found an equation to calculate the ability of the soil we filled in the columns to conduct water. I would like to call this ability "hydraulic conductivity", but I am yet unsure about this.

Let me explain, what we found. After several experiments with different columns that were filled with the different kinds of soil, I would like to postulate that the flow rate is proportional to the water level difference, the dimensions of the columns, and the properties of the soil; thus, I think we can say that

$$Q = -k_f \cdot A \cdot \frac{\Delta h}{L} \tag{1}$$

As an example, I added a sketch of the first experiment (figure 2). We have measured the length of the column $L = 150\,\mathrm{cm}$, and its the diameter $d = 22\,\mathrm{cm}$, the water levels at the in and outflow tubes as $h_1 = 40\,\mathrm{cm}$ and $h_2 = 37\,\mathrm{cm}$, respectively (we always used the surface of the table where the apparatus is set up as the reference level), producing a water level difference $\Delta h = h_2 - h_1$. We also used a bucket to measure the flow rate through the column as $Q = 0.05\,\mathrm{l} \cdot \mathrm{min}^{-1}$. With this, I would categorize the soil's hydraulic conductivity through the following set of equations. The difference of the water levels is

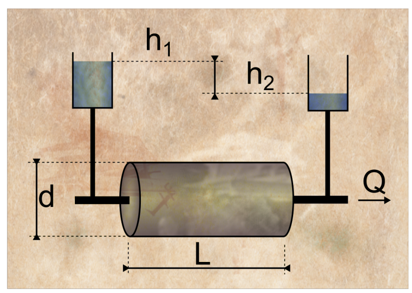

Figure 2: Experiment 1

$$\Delta h = h_2 - h_1 = 0.37\,\mathrm{m} - 0.4\,\mathrm{m} = -0.03\,\mathrm{m} \tag{2}$$

and the hydraulic head gradient

$$i = \frac{\Delta h}{L} = \frac{-0.03\,\mathrm{m}}{1.5\,\mathrm{m}} = -0.02\,. \tag{3}$$

I know the formula to calculate the area of a circle (inner of the column) to be

$$A = 0.25 \cdot \pi \cdot d^2 = 0.038\,\mathrm{m}^2, \tag{4}$$

and I understand that a total volume $V = 1\,\mathrm{m}^3$ of water is equal to as many as $1000\,\mathrm{l}$. Therefore, we should get the hydraulic conductivity with the flow area of the column

$$k_f = -\frac{Q}{A \cdot i} = -\frac{0.05 \cdot 10^{-3}\,\mathrm{m}^3 \cdot \mathrm{min}^{-1}}{0.038\,\mathrm{m}^2 \cdot (-0.02)} = 0.066\,\mathrm{m} \cdot \mathrm{min}^{-1}. \quad (5)$$

DIY:

1. *Can you convert the result of k_f into units of $\mathrm{m} \cdot \mathrm{s}^{-1}$?*
2. *What kind of soil did the researchers use (sand, silt, clay...)?*

Entry from August, 20th, 1857. After the successful first experiment, here is the second setup we used to investigate a different soil type (figure 3). This time, we filled the column with a soil of finer grains than the one we used before. Again, we ensured, that the filling did not produce any heterogeneities in the column (e.g. layers or air bubbles).

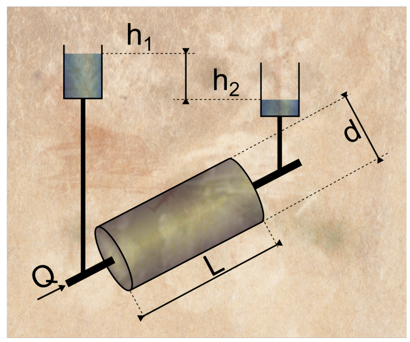

Figure 3: Experiment 2

We measured $L = 8\,\mathrm{dm}$, $d = 3\,\mathrm{dm}$, $Q = 6\,\mathrm{l} \cdot \mathrm{h}^{-1}$, $h_1 = 52\,\mathrm{cm}$, and $h_2 = 44\,\mathrm{cm}$. I am still puzzled about the fact, that the water flows upwards! I am wondering, if the postulated formula also works for this setup...

DIY:

1. *Calculate the hydraulic conductivity k_f for this second example!*
2. *What kind of soil did the researchers use in this experiment (sand, silt, clay...)?*

Entry from August, 21st, 1857. In one of the experiments, I did a mistake and measured the wrong distance $L^\star$ as the distance between the stand pipes (figure 4). We used the same soil as in experiment 1, but a different column with $d = 0.2\,\mathrm{m}$, $\Delta h = 0.8\,\mathrm{cm}$, and $Q = 0.34\,\mathrm{l} \cdot \mathrm{min}^{-1}$. I need to find the length of the column L (which I remember was pretty long), because I want to reproduce the experiment - but how?

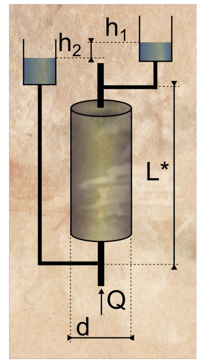

Figure 4: Exp. 3

> **DIY:**
>
> 1. *Guess what... find the length of the column L!*
> 2. *Why do you think would one like to repeat an already successful experiment?*

This all may be a small step for us, but it is certainly a giant leap for mankind.

H.D.